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LETTER TO THE EDITOR

Existence of a spin liquid state for the frustrated quantum Heisenberg antiferromagnet for large values of the spin

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Abstract. The existence of a spin liquid ground state for the strongly frustrated quantum antiferromagnet on a square lattice is reconsidered. Spin wave theory shows that the stability of the Néel state is increased along the classical critical line except at the Lifshitz point, while that of the spiral state is depressed by the same amount. The system is described by a unique action which is continuous through the transition point. A renormalization group analysis shows that the effective coupling constant flows towards strong coupling in its vicinity.

The recent surge of interest in 2+1-dimensional magnetism has focused on the role of quantum fluctuations, frustration and topology [1–3] in the properties of the zero-temperature ground state of spin systems. It has been seen in particular that their combined effect can induce spin disordered states and new kinds of order: valence bond solid [4], spin Peierls [5], chiral [6] or spin nematic [7].

The aim of this letter is to shed some light on the recent controversy about the existence of a spin liquid ground state for the J_1–J_2–J_3 quantum antiferromagnet, for large values of the spin and close to the classical transition line between the Néel and spiral ordered states. It is striking that, despite intense efforts, the problem is still under discussion, linear spin wave theory (LSWT) [8], numerical diagonalization [9, 10], finite series expansions [11] and renormalization group analysis [12] giving a spin liquid state and the diverse large-N or other self-consistent theories [5, 13–17] theories giving *order from disorder* [18] and a first-order phase transition between the ordered states, instead. The main points below will be as follows.

(i) The *classical critical line*, CCL, $J_1-2J_2-4J_3=0$, separates states with the same symmetry. The transition is continuous classically, in the sense that the parameters of the long-wavelength action $S[J_i]$ vary smoothly along the transition. The same happens for the quantum case, $S[J_i, S]$: although there is an enhancement of the stability of the Néel state close to the CCL, there is an identical decrease of the stability of the spiral state. This means that the CCL is continued in a *quantum critical plane*, QCP, which depends on S and is tilted towards bigger values of J_2 and J_3.

(ii) Because the quantum action is continuous through the QCP, a generalized sigma model can be used to describe the physics of the transition along the QCP

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and at its sides. A renormalization group analysis shows that the relevant coupling constant always flows to strong coupling, meaning that the spin state is disordered. The spin liquid state exists not only along that plane but also in a finite region around it.

(iii) The end point of the CCL is a Lifshitz point. In this case, the counterpart of the Néel state is not spiral, but collinear. Because the two states have different symmetry, the classical actions for the two states are different, and there is no continuity principle.

One can write the most general action for a classical helical magnet by noting that the order parameter space is $O(3) \times U(1)/U(1)$ [7, 19, 20]

$$S = -\frac{1}{2}\int d^2x \, \mathrm{Tr}\{A_i P_i A_i\} = -\frac{1}{2}\int d^2x \, \rho^a (A_i^a)^2 \tag{1}$$

where $A_i = A_i^a T_a = g^{-1}\partial_i g$ is a pure gauge field, $g(x) \in SO(3)$ and $T_a \in$ Lie[SO(3)]. The gauge field is equivalent to a twist of the order parameter and serves to define the spin stiffnesses, $\rho^a = (\rho^x, \rho^x, \rho^z)$. The action $S[\rho(Q_{0,\mathrm{cl}})]$ is unique for both Néel and helical states and the spin stiffnesses are continuous throughout the whole phase diagram. $Q_{0,\mathrm{cl}}$ is the pitch wavevector of the classical ground state.

After introducing quantum fluctuations, there is still a unique action for the whole phase diagram, which varies continually when passing from the Néel to the spiral state through the *quantum critical plane*. The action picks up an extra piece, due to the fact that the fields become dynamical, and an implicit dependence on S through $Q_0(S)$—the pitch of the quantum ground state

$$S = -\frac{1}{2}\int dx_0 \, d^2x \, (\chi^a (A_0^a)^2 + \rho^a (A_i^a)^2). \tag{2}$$

It will be proven below using spin wave theory (SWT) that the effect of quantum fluctuations is: (1) to enhance the spin stiffness of the Néel state and reduce by the same amount that of the spiral state so that both go to zero at the same displaced point; (2) to renormalize also the pitch wavevector of the helical state so as to adjust smoothly to the new boundary of the Néel state, where its value is (π, π).

The *order from disorder* conjecture can also be proven using SWT by computing the expression for the staggered magnetization close to the classical frustrated point:

$$\langle S^z \rangle \simeq S + \frac{1}{2} - \alpha \ln(\rho_{\mathrm{cl}}) + \frac{\beta}{S}\frac{1}{\rho_{\mathrm{cl}}} + O\left(\frac{1}{S^2}\right) \tag{3}$$

where $\rho_{\mathrm{cl}} = J_1 - 4J_3$ is the classical spin stiffness—divided by S^2—which is used as a cut-off for the infrared divergent integrals [21]. This series can be resumed: computing the spin wave energy to next to LSWT order in $1/S$, one finds:

$$\langle S^z \rangle = S + \tfrac{1}{2} - \gamma \ln(\rho(S)) + O(1/S) \tag{4}$$

$$\rho(S) = \rho_{\mathrm{cl}} - \frac{4J_3}{S}\sum_k \frac{A(k)\gamma(2k) - \gamma^2(k)}{\sqrt{A^2(k) - \gamma^2(k)}} + O(1/S^2)$$

$$\gamma(k) = \tfrac{1}{2}(\cos(k_x) + \cos(k_y)) \qquad A(k) = 1 - J_3(1 - \gamma(2k))$$

which is the LSWT result with a renormalized spin stiffness. It serves us to define the QCP as the plane where $\rho(S)$ is zero. That plane is tilted to the right of the CCL.

The QCP has to be the same for both the helicoidal and the Néel states (at least for large S). In other words, the enhancement of the stability of the Néel state is accompanied by a similar reduction of the stability of the helicoidal one so that both states match each other continuously. It is sufficient to compute the LSWT correction to the pitch wavevector and to the stiffness to prove it:

$$q_0^i - q_{0,\mathrm{cl}}^i = \Delta q_0^i = \frac{1}{2S\,\partial_i^{(2)} J_{Q_{0,\mathrm{cl}}}} \sum_k \partial_i J_\pm \left(\frac{J_k - J_{Q_{0,\mathrm{cl}}}}{J_\pm - J_{Q_{0,\mathrm{cl}}}}\right)^{1/2} + \mathrm{O}\left(\frac{1}{S^2}\right)$$

$$\rho_{ii}^z = \frac{S^2\,\partial_i^{(2)} J_{Q_{0,\mathrm{cl}}}}{2} + \frac{S}{2}\Big\{\partial_i^{(2)} J_{Q_{0,\mathrm{cl}}} - (\sin(q_{0,\mathrm{cl}}) + 8J_3\sin(2q_{0,\mathrm{cl}}))S\,\Delta q_0^i$$
$$+ \frac{1}{2}\sum_k \frac{(2J_{Q_{0,\mathrm{cl}}} - J_k - J_\pm)\,\partial_i^{(2)} J_{Q_{0,\mathrm{cl}}} - (J_{Q_{0,\mathrm{cl}}} - J_k)\,\partial_i^{(2)} J_\pm}{\sqrt{A^2(k) - \gamma^2(k)}}\Big\} + \mathrm{O}(1) \tag{5}$$

where J_k is the Fourier transform of J_{ij} and $J_\pm = (J_{k+Q} + J_{k-Q})/2$. The spin stiffness of the unfrustrated magnet is $\rho = \rho_{\mathrm{cl}}(1 - 0.118/S)$ ($= 0.764\rho_{\mathrm{cl}}$ for spin 1/2).

Figure 1(a) is a plot of the LSWT correction to q_{cl} for the spiral state to show that it matches with (π, π) continually. Figure 1(b) is a plot of the LSWT correction to the spin stiffness, and shows that it is enhanced for the Néel state close to the critical point and it is decreased by the same amount on the other side. Although we have not gone further, *we conjecture that these two phenomena happen at every order in the perturbative 1/S expansion, so that the action is continuous and unique.*

Once it is proven that the physics of the system is continuous through the transition point, one can look for the explicit form of the action $\mathcal{S}[\rho(Q_0)]$ which describes the magnet close to the transition point—from both sides—and perform a renormalization group study. It is a generalization of the $\mathrm{O}(3)/\mathrm{U}(1) = S^2$ quantum non-linear σ model [22]. Close to the QCP the bare spin stiffness, $\rho(S)$, is very small and one has to take into account effects due to quartic terms. The zero-temperature action is

$$\mathcal{S} = \frac{1}{2}\int_{-\infty}^{\infty}\int_a^L \mathrm{d}t\,\mathrm{d}^2r \Big\{\chi^0(\partial_t\hat{n})^2 - \rho^0(S)((\partial_x\hat{n})^2) + (\partial_y\hat{n})^2))$$
$$+ \frac{\sigma_1^0 a^2}{12}((\partial_{xx}\hat{n})^2 + (\partial_{yy}\hat{n})^2) - \sigma_2^0 a^2(\partial_{xx}\hat{n})(\partial_{yy}\hat{n})\Big\}. \tag{6}$$

$\rho^0(S)$ is given by SWT (i.e. the stiffness renormalized by 'short'-wavelength quantum fluctuations), while σ_i^0 have the values obtained in a gradient expansion. The naive charge is $g_0 = 1/\sqrt{\rho\chi}a$.

One of the problems which arise in the scaling analysis of the frustrated magnet is that the coupling constant g_0 depends not only on S (as in the unfrustrated case), but also on the exchange interactions J_1, J_2 and J_3. Therefore, there is not a clear indicator of the magnetic order of the microscopic model. Moreover, g_0 diverges at the QCP, and therefore is not well suited for our analysis.

After performing the perturbative RG expansion [23–25] one arrives at the following scaling equations:

$$\begin{aligned} \mathrm{d}g/\mathrm{d}l &= -g + g^2 I\,(1 + (\sigma_1 - 2\sigma_2)/8\rho) \\ \mathrm{d}\rho/\mathrm{d}l &= -\rho g I\,(1 + (\sigma_1 - 2\sigma_2)/4\rho) \\ \mathrm{d}\sigma_i/\mathrm{d}l &= -\sigma_i(2 + gI) \end{aligned} \tag{7}$$

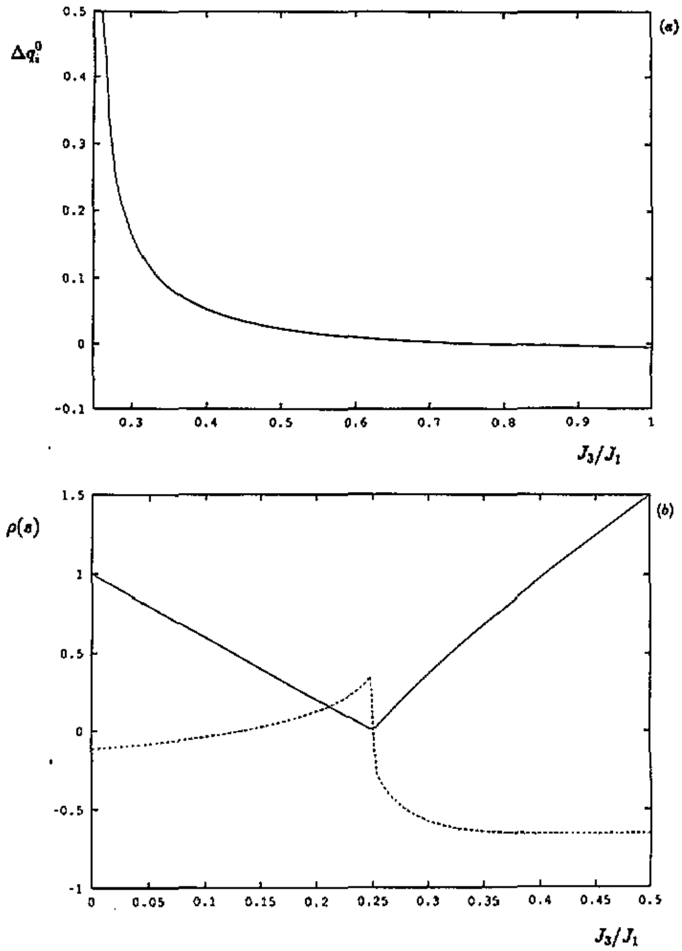

Figure 1. (*a*) LSWT correction to $q_{0,\mathrm{cl}}$; (*b*) classical spin stiffness (solid lines) and LSWT corrections to it (dashed lines)

where I is the loop integral

$$I = \frac{\sqrt{6\rho}}{\pi^2} \int_0^{\pi/2} \frac{\mathrm{d}\theta}{\sqrt{24\rho - 2\sigma_1 + (\sigma_1 + 6\sigma_2)\sin^2\theta}}. \tag{8}$$

This integral diverges at the Lifshitz point, signalling that the scaling analysis breaks down there: this point is quite pathological, and has to be treated in a different way from the rest of the phase diagram. These scaling equations reduce to those obtained from the weakly frustrated case [23] and to those obtained in [11] when $\rho(S) = 0$ [12].

An unambiguous indicator for the ordering of the system is the charge

$$G^0 = \frac{I}{\sqrt{\rho(S)}\chi a} = \frac{A}{\chi^{1/2}\sigma_1^{1/4}\sigma_2^{1/4}a} \sim \sqrt{\frac{J_1}{(J_2 J_3)^{1/2}}}\frac{1}{S} \qquad (\rho^0 = 0) \tag{9}$$

introduced by Ioffe and Larkin [12]. Assuming that $A = (\sigma_1\sigma_2)^{1/4}I/\rho^{1/2}(S)$ does not get renormalized, it is found that G satisfies

$$\mathrm{d}G/\mathrm{d}l = G^2 \tag{10}$$

which is the effective charge of a system at its lower critical dimension, 1+1 here. It flows towards strong coupling and generates a correlation length $\xi \sim a\mathrm{e}^{1/G_0} \sim a\mathrm{e}^{S}$.

To draw the separatrix between the ordered state and the spin liquid we follow Ioffe and Larkin and argue that for finite $\rho(S)$ the relevant action is not given by equation (6), but by

$$\mathcal{S} = \frac{1}{2}\int_{-\infty}^{\infty}\int_{\ln(1/\rho(S))}^{L} \mathrm{d}t\,\mathrm{d}^2r \left\{\chi^0(\partial_t\hat{n})^2 + \frac{\sigma_1^0 a^2}{12}((\partial_{xx}\hat{n})^2 + (\partial_{yy}\hat{n})^2) - \sigma_2^0 a^2(\partial_{xx}\hat{n})(\partial_{yy}\hat{n})\right\} \tag{11}$$

where $\ln(1/\rho(S))$ acts as a cut-off in the scaling equations. In that case, the separatrix is given by

$$1 + G_0\ln(\rho(S)) = 0. \tag{12}$$

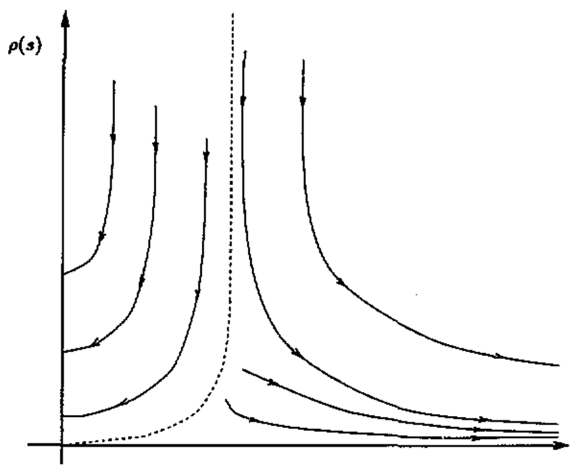

Figure 2. Flow diagram for the J_1–J_2–J_3 model, linking the weakly and the strongly frustrated magnets. We plot on the y-axis $\rho(S)$, the spin stiffness renormalized by short-wavelength quantum fluctuations. Rotating the figure by 90° we obtain the phase diagram of SWT.

It smoothly joins the one for the weakly frustrated case [26] (see figure 2). It is amusing to note that on turning the figure by 90°, the diagram obtained is the same as the one given by SWT (and Schwinger bosons mean field theory), if one takes into account that $\rho(S)$ is the dressed spin stiffness.

In conclusion, the J_1–J_2–J_3 model has a spin disordered phase shifted from the CCL, because (1) the same fluctuations which enhance the stability of the Néel state depress that of the spiral state; (2) the scaling analysis for the strongly frustrated magnet shows that the system flows towards strong coupling.

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